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Bianchi Type VI String Cosmological Model in Saez–Ballester's Scalar-Tensor Theory of Gravitation

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Abstract An exact Bianchi type-VI string cosmological model is obtained in a scalar-tensor theory of gravitation proposed by Saez and Ballester (Phys. Lett. 113:467, 1985). Some physical properties of the model are also discussed.

Keywords Cosmic strings · Saez–Ballester's scalar-tensor theory · Bianchi type VI model · Inflationary model

1 Introduction

In recent years there has been a lot of interest in the study of cosmic strings and string cosmological models in general relativity and also in new theories of gravitation. Cosmic strings are the topological defects associated with spontaneous symmetry breaking whose plausible production site is cosmological phase transitions in the early universe [1].

The gravitational effects of cosmic strings have been extensively discussed by Satchel [5], Vilenkin [2], Latelier [4] and Gott [3] in general relativity. So far a considerable amount of work has been done on cosmic strings and string cosmological models by Krori et al. [6], Banerjee et al. [7], Tikekar and Patel [8], Tikekar et al. [9], Bhattacharjee and Baruach [11].

Recently a lot of work has been done on cosmic strings and string cosmological models. Mahanta and Mukherjee [16], Rahaman et al. [17, 18] and Reddy [19] are some of the authors who have investigated several aspects of cosmic strings in scalar tensor theory based on Lyra geometry. Gundlack and Ortiz [22], Barros and Romero [21], Sen et al. [23] and Sen [24], Barros et al. [20] have discussed interesting string cosmological models in Brans–Dicke scalar tensor theory of gravitation. Recently, Reddy [25, 26] has obtained a Bianchi type I string cosmological models in Brans–Dicke theory and in the new scalar-tensor theory formulated by Saez–Ballester [10].

In the last few decades there has been considerable interest in alternative theories of gravitation. The most important among them being scalar-tensor theories proposed by Lyra [12],

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Brans and Dicke [13], Nordvet [14], Wagoner [15], Saez and Ballester [10] have developed a theory in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of weak fields.

In this paper, the field equations in the scalar tensor theory developed by Saez and Ballester [10] are considered for the Bianchi Type VI metric in the presence of cosmic string source. An exact solution of the field equations is obtained. Some physical properties of the cosmological model are also discussed.

2 Field Equations and the Cosmological Model

The field equations in the new scalar-tensor theory proposed by Saez and Ballester [10] are

$$G_{ij} - \omega\phi^n\left(\phi_{;i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{'k}\right) = -T_{ij} \tag{1}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n\phi_{;i}^{;i} + n\phi'\phi_{,k}\phi^{'k} = 0, \tag{2}$$

where G_{ij} is the Einstein tensor, ω and n are constants. T_{ij} is the stress tensor of matter.

Comma (,) and Semicolon (;) denote partial and covariant differentiation respectively.

Also

$$T^{ij}_{;j} = 0 \tag{3}$$

is consequence of the field equations (1) and (2).

We consider the spatially homogeneous Bianchi type-VI metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2qx}dy^2 + C^2e^{2qx}dz^2, \tag{4}$$

where A, B, C are the functions of t only and q is a non zero constant.

The energy momentum tensor for cosmic strings is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j. \tag{5}$$

Here ρ = rest energy density of cloud of strings with particles attached to them, λ = tension density of strings, u^i = cloud four-velocity, x^i = direction of anisotropy.

We have

$$u^i u_i = -1, \qquad x_i x^i = 1, \qquad u^i x_i = 0. \tag{6}$$

We consider

$$\rho = \rho_p + \lambda,$$

where ρ_p is the rest energy density of particles and x^i to be along X-axis, so that

$$x^i = (0, A^{-1}, 0, 0). \tag{7}$$

Here the scalar field ϕ is function of the cosmic time t only.

The field equations (1), (2) and (3) for metric (4) with the help of (5), (6) and (7) can be written as

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4}{B}\frac{C_4}{C}+\frac{q^2}{A^2}=\lambda+\frac{\omega}{2}\phi^n\phi_4^2, \tag{8}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4}{A}\frac{C_4}{C}-\frac{q^2}{A^2}=\frac{\omega}{2}\phi^n\phi_4^2, \tag{9}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4}{A}\frac{B_4}{B}-\frac{q^2}{A^2}=\frac{\omega}{2}\phi^n\phi_4^2, \tag{10}$$

$$\frac{A_4}{A}\frac{B_4}{B}+\frac{B_4}{B}\frac{C_4}{C}+\frac{A_4}{A}\frac{C_4}{C}-\frac{q^2}{A^2}=\rho-\frac{\omega}{2}\phi^n\phi_4^2, \tag{11}$$

$$\frac{B_4}{B}-\frac{C_4}{C}=0, \tag{12}$$

$$\phi_{44}+\phi_4\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)+\frac{n}{2}\frac{\phi_4^2}{\phi}=0, \tag{13}$$

$$\rho_4+(\rho-\lambda)\frac{A_4}{A}+\left(\frac{B_4}{B}+\frac{C_4}{C}\right)=0, \tag{14}$$

where suffix 4 indicates differentiation with respect to t.

Equation (12) readily gives

$$B=\mu C,$$

where μ being an integration constant. Without loss of generality, we take $\mu=1$.

Hence, we have

$$B=C. \tag{15}$$

Using the transformations

$$A=e^{\alpha},\qquad B=e^{\beta},\qquad dt=AB^2dT$$

(8–14) reduce to

$$2\beta''-2\alpha'\beta'-(\beta')^2+q^2e^{4\beta}-\frac{\omega}{2}\phi^n(\phi')^2=\lambda\exp[2(\alpha+2\beta)], \tag{16}$$

$$\alpha''+\beta''-2\alpha'\beta'-(\beta')^2-q^2e^{4\beta}-\frac{\omega}{2}\phi^n(\phi')^2=0, \tag{17}$$

$$2\alpha'\beta'+(\beta')^2-q^2e^{4\beta}+\frac{\omega}{2}\phi^n(\phi')^2=\rho\exp[2(\alpha+2\beta)], \tag{18}$$

$$\phi''+\frac{n}{2}\frac{(\phi')^2}{\phi}=0, \tag{19}$$

$$\rho'+(\rho-\lambda)\alpha'+2\rho\beta'=0, \tag{20}$$

where a dash denotes differentiation with respect to T.

In view of the highly non-linear character of the field equations (16–20), we obtain an exact solution of field equations when the sum of rest energy density and the tension density of the cloud of string vanishes, i.e. when

$$\rho + \lambda = 0. \tag{21}$$

This case is found to be physically interesting case as it narrates the possibility of our solution being of the inflationary type for string cosmological models.

The exact solution of the field equations (16–20) in this case, is given by

$$A = \exp\left[\frac{k_1 \exp[2(aT+b)]}{4a^2} + \frac{q^2 \exp[4(aT+b)]}{8a^2} + k_2 T + k_3\right], \tag{22}$$

$$B = C = \exp(aT+b), \tag{23}$$

$$\phi = \left[\left(\frac{n+2}{2}\right)(k_4 T + k_5)\right]^{\frac{2}{n+2}}, \tag{24}$$

$$\begin{aligned} \rho &= -\lambda \\ &= k_6 \exp\left[-2\left[\frac{k_1 \exp(aT+b)}{4a^2} + \frac{q^2 \exp[4(aT+b)]}{8a^2} \right.\right. \\ &\quad \left.\left. + k_2 T + k_3 + \exp(aT+b)\right]\right], \end{aligned} \tag{25}$$

where k_1, k_2, k_3, k_4, k_5, k_6 are constants of integration.

After a proper choice of coordinates and constants, the corresponding metric can be written in the form

$$\begin{aligned} ds^2 = {} & -\exp(6T)dt^2 + \exp 2\left[\frac{k_1 \exp(2T)}{4a^2} + \frac{q^2 \exp(4T)}{8a^2} + k_2 T + k_3\right]dx^2 \\ & + \exp 2(T - qx)dy^2 + \exp 2(T + qx)dz^2. \end{aligned} \tag{26}$$

3 Physical Properties

The model represented by (26) represents spatially homogeneous Bianchi type VI inflationary string cosmological model in the scalar-tensor theory of gravitation proposed by Saez and Ballester [10]. The Scalar field ϕ, the rest energy density and the tension density of the strings in the model are given (24) and (25) respectively. It should be noted that the model, the rest energy density and the tension density of the cloud of strings as well as the scalar field have no initial singularity.

The spatial volume of model (26) is

$$\begin{aligned} V &= (-g)^{1/2}, \\ V &= \exp\left[\frac{2k_1 \exp(2T) + q^2 \exp(4T)}{8a^2} + (k_2 + 5)T + k_3\right]. \end{aligned} \tag{27}$$

The expansion scalar

$$\theta = \frac{1}{3}u^i_{;i} = \frac{1}{3}\left[\frac{k_1 \exp(2T) + q^2 \exp(4T)}{2a^2} + k_2 + 5\right]. \tag{28}$$

The shear scalar

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{7}{162}\left[\frac{k_1\exp(2T) + q^2\exp(4T)}{2a^2} + k_2 + 5\right]. \tag{29}$$

For large values of T, we get

$$\lim_{t\to\infty}\frac{\sigma}{\theta} = -3\sqrt{\frac{7}{162}} \neq 0. \tag{30}$$

The deceleration parameter

$$\begin{aligned} q &= -3\theta^{-2}\left[\theta_{;\alpha}u^{\alpha} + \frac{1}{3}\theta^2\right] \\ &= \frac{-9}{a^2}\left[\frac{k_1\exp(-T) + 2q^2\exp(T)}{[\frac{k_1\exp(2T)+q^2\exp(4T)}{2a} + (k_2+5)]^2}\right] \\ &= -36\left[\frac{k_1\exp(-T) + 2q^2\exp(T)}{[k_1\exp(2T) + q^2\exp(4T) + 2a(k_2+5)]^2}\right]. \end{aligned} \tag{31}$$

The Hubble parameter

$$H = \frac{1}{3}\theta = -\left[\frac{k_1\exp(2T) + q^2\exp(4T)}{2a^2} + k_2 + 5\right]. \tag{32}$$

4 Discussion

From (27), as the time increases the spatial volume also increases which implies the anisotropic expansion of universe with time.

From equation (30), $\lim_{t\to\infty}\frac{\sigma}{\theta} \neq 0$, the model do not approach isotropy for large values of T.

From equation (31), it is observed that q turns out to be negative, which confirms the fact that model (26) represents inflation [27].

5 Conclusion

Here we have presented a spatially homogeneous Bianchi type VI string cosmological model in the scalar-tensor theory proposed by Saez and Ballester [10]. The model, thus obtained, is found to be inflationary type and free from initial singularity. The scalar field and density are also free from initial singularity. The model is anisotropic and is expanding with time.

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